ANNEX I. INITIAL VERSION OF THE QUESTIONNAIRE (v1)

Item 1

Let us consider the plane $H \coloneqq \{(x, y) \in R^2 \text{ such that } y > 0\}$. The following sets are regarded as lines:

- Vertical lines.
- Circles with centers in $\{(x, 0)\} \cap H$.

Construct a definition of parallel lines such that if a, b, and c are lines and ∥ denotes the relation of being parallel, then a∥b and b∥c implies a∥c. That is, with this definition, the relation "being parallel lines" is transitive. What is the significance of having "being parallel lines" as a transitive relation?

Item 2: Let us consider the plane

$H \coloneqq \{(x, y) \in R^2 \text{ such that } y > 0\}$. The following sets are regarded as lines:

- Vertical lines.
- Circles with centers in $\{(x, 0)\} \cap H$.

Given this, the following two definitions of parallel lines are proposed:

Def1: Two vertical lines are considered parallel, and two circles are parallel if they share the same center. A vertical line is never parallel to a circle.

Def2: Two lines are considered parallel if they share the same center (understood as all vertical lines having the same center, which is taken to be a point not in R^2).
Question 1: Which of the two definitions do you consider more appropriate, and why?
Question 2: Which of the two definitions would you use to determine which of these lines are parallel?

- x=7
- x=10
- $(x-5)^2 + y^2 = 20$
- $(x-5)^2 + y^2 = 30$
- $(x-3)^2 + y^2 = 100$

Item 3

Normally, to measure distances in the plane, we use the Euclidean distance, which is defined as follows: given points A=(x_1,y_1) and B=(x_2,y_2), the distance between them is given by the length of the line segment that connects them:

$$d_E(A, B) := \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

However, we can define other different distances, such as the **radial distance**, which is defined as follows:

$$d_R(A, B) \coloneqq \{d_E(A, B) \text{ if } A, B \text{ and } O \text{ are aligned } d_E(A, O) + d_E(O, B) \text{ otherwise}$$

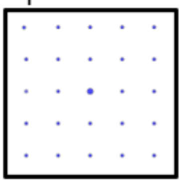
Here, O represents the origin of coordinates, i.e., the point with coordinates O=(0,0). If we define a circle as the set of points equidistant (at the same distance) from another point, **what shape do the circles have with the radial distance**?

Item 4

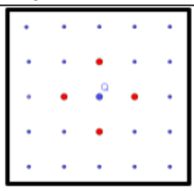
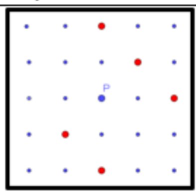
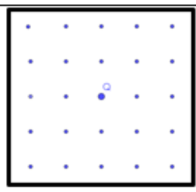
Normally to measure distances in the plane we use the Euclidean metric, which is defined as follows: Given two points, the distance between them is the length of the segment joining them. However, we can define other distances, such as the so-called postman's (or **Taxicab**) distance, which is defined as follows: the distance between two points is given by the shortest route joining those using only horizontal and vertical lines.

Item 4.1: If we define the circumference as the set of points that are equidistant (at the same distance) from another point, what is the shape of the circumferences with the Taxicab metric? Justify your answer.

Item 4.2: If we define a circumference as a set of points that are equidistant from another point, what shape do they have with the distance from the postman? Justify your answer. You can use the grid to draw the points that are equidistant from the point indicated.



Item 4.3: If we define a circle as a set of points that are equidistant (at the same distance) from another point. Look at the different drawings below:

All the marked points are, with the Taxicab metric, at distance __ from point Q.	All the marked points are, with the Taxicab metric, at distance __ from point P.	What is the shape of the circles with the Taxicab metric? Justify your answer.
		

ANNEX II. FINAL VERSION OF THE QUESTIONNAIRE (v4)

ITEM 1 Let us consider the plane $H := \{(x, y) \in R^2 \text{ such that } y > 0\}$ and consider the straight lines as the following sets:

- $V_a = \{(a, b) | b \geq 0;\ a, b \in R\}$
- The intersection of H with any circumference with center in $(a, 0)$.

Item 1.1 Construct a definition of parallel straight lines in the plane H such as if a, b and c are straight lines and we denote by || the relation of being parallel, it satisfies that if a || b and b || c then a || c. That is, construct a transitive definition of parallelism.

Can you think of a reason why it might be necessary to define the parallelism relationship in this particular manner?

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Item 1.2 Consider the following equivalent definitions of parallelism:

Definition 1: Two sets V_a and V_b are always considered parallel. Two (semi)circumferences are considered parallel only if they share the same center. A vertical line is never parallel to any semi circumference.

Definition 2: Two straight lines are considered parallel if they share the same center (assuming that every set of the type V_a has the same center, which is a point not belonging to R^2).

From the two previous definitions, which one would you select in order to determine which pairs of the following straight lines are considered parallel?

r: x=7

s: x=10

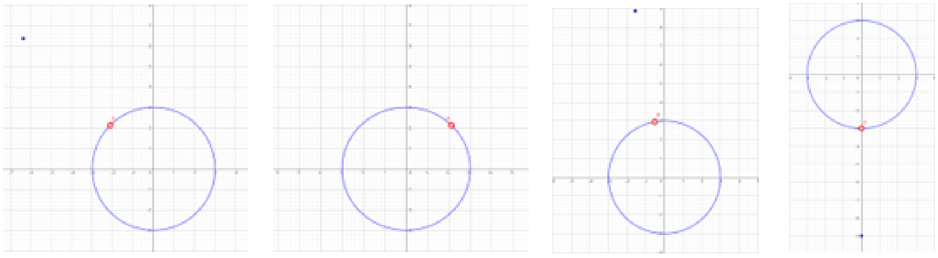
t: $(x-5)^2 + y^2 = 20$

u: $(x-5)^2 + y^2 = 30$

v: $(x-3)^2 + y^2 = 100$

ITEM 2

Item 2.1 In the given images, the blue points share a property with respect to the red point (A).



Task 1: Construct a definition for the set of blue points -above- with respect to the red point (A). We will call this set S_A

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Normally, to measure distances in the plane, we use the Euclidean distance, which is defined as follows. Given two points A=(x_1,y_1) and B=(x_2,y_2), the distance between them is given by the length of the line segment that connects them, that is,

$$d_E(A,B) := \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

However, we can define other different distances, such as the **radial distance**, which can be defined as follows:

Definition 1:

$$d_R(A,B) := \begin{cases} d_E(A,B) \text{ if } A, B \text{ and } O \text{ are aligned} \\ d_E(A,O)+d_E(O,B) \text{ otherwise} \end{cases}$$

Here, *O* represents the origin of coordinates, that is, the point with coordinates *O*=(0,0).

Task 2. Construct a definition for the set of blue points -above- with respect to the red point (A) using the radial distance. We will call this set S_A using Definition 1.

Task 3: Review your responses to tasks 1 and 2. Can you think of a reason why Definition 1 might be necessary?

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Item 2.2 Similarly, the radial distance can be described as follows:

Definition 2: The radial distance between two points is the minimum length of the segment or segments that connect them, with these segments contained in lines that pass through the origin.

Compare definitions 1 and 2 by choosing which one you would use in the following cases:

-To present the metric informally.

-To measure distances between specific points.

ITEM 3 Normally to measure distances in the plane we use the Euclidean metric, which is defined as follows: Given two points, the distance between them is the length of the segment joining them. However, we can define other distances, such as the so-called postman's (or Taxicab) distance, which is defined as follows: the distance between two points is given by the shortest route joining them using only horizontal and vertical lines.

Item 3.1 If we define the circumference as the set of points that are equidistant (at the same distance) from another point, what is the shape of the circumferences with the Taxicab metric? Justify your answer.

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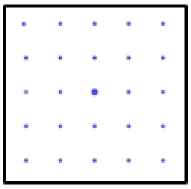
Item 3.2 If we define the circumference as the set of points that are equidistant from another point, what shape do they have with the Taxicab distance? Justify your answer. You can use the grid to draw the points that are equidistant from the point indicated.



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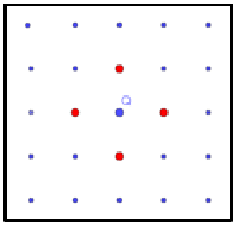
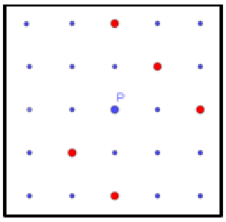
Item 3.3 If we define the circumference as the set of points that are equidistant (at the same distance) from another point. Look at the different drawings below:

All the marked points are, with the Taxicab metric, at distance __ from point Q.



All the marked points are, with the Taxicab metric, at distance __ from point P.



What is the shape of the circles with the Taxicab metric? Justify your answer.